CONSTRUCTION AND ANALYSIS OF DESIGNS $q \times 3^2$ IN SIX BLOCKS

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Introduction

The construction and analysis of asymmetrical factorial designs still remains a problem. Though, several methods of construction of these designs have been developed, all these attempts have been to obtain balanced designs, which require large amount of resources. In this paper, a method of construction (along with analysis) of designs $q \times 3^2$ using only six blocks, wherein the main effects and interaction effects can be estimated mutually independently, as pointed out by Sardana and Das (1965), has been presented. The construction of generalised design $q \times 3^n$ has also been indicated.

2. Construction of Designs $q \times 3^2$ in six blocks

Let A, B and C be the three factors at levels q, 3 and 3 respectively. The q levels of the factor A could be of any of the three forms $3k$, $(3k-1)$ and $(3k-2)$, where k is a positive integer. For constructing the design, we first bring the levels of factor A to $3k$ by repeating one or two levels of A, when q is not of the form $3k$. The levels of A, thus increased to $3k$, can be considered to be combinations of two pseudo factors X and Y at levels k and 3 respectively. Then the problem reduces to that of constructing the design $k \times 3^3$ in six blocks each of $9k$ plots. This is done by partially confounding YBC and YB^2C^2 each with 2 d.f. in two replications of $k \times 3^3$ with X, Y, B and C as the factors, and then replacing back the treatment combinations of X and Y by levels of A. The different schemes of such replacements for the three forms of q are given in Table 1.

3. Analysis of Designs $q \times 3^2$ in six Blocks

It can be seen that in all the six types of designs obtained in section 2, the main effects and two factor interactions are not affected by block differences, since all the two factor treatment combinations occur in each block equal number of times. Thus the independent estimates of these main effects and two factor interactions and the sums of squares (S.S.) due to them can be obtained in the usual manner.

It can also be observed that the interaction $A(BC^2)$ with $2(q-1)$ d.f. is also not affected in these designs since every level of A occurs in each block with the same number of treatment combinations of B and C from each of the three sets *viz.*, (b_0c_0, b_1c_1, b_2c_2), (b_0c_2, b_1c_0, b_2c_1), (b_0c_1, b_1c_2, b_2c_0), which lead to the interaction BC^2, replicated equal number of times. Hence the S.S. and independent estimates of effects due to $A(BC^2)$ can also be obtained in the usual manner.

Thus, only $A(BC)$ with $2(q-1)$ d. f. could be affected by block differences in these designs. This interaction can be split into $(q-1)$ components, each with 2 d. f., corresponding to the $(q-1)$ contrasts of the main effect A. For example, if $\Sigma_i w_{ji} a_i$ is the *j-th* contrast of the main effect A, then the *j-th* component of $A(BC)$ will be due to the two comparisons between $\Sigma_i w_{ji} (a_i b_0 c_0 + a_i b_1 c_2 + a_i b_2 c_1)$, $\Sigma_i w_{ji} (a_i b_0 c_1 + a_i b_1 c_0 + a_i b_2 c_2)$ and $\Sigma_i w_{ji} (a_i b_0 c_2 + a_i b_1 c_1 + a_i b_2 c_0)$. Out of these $(q-1)$ components, thus defined, only two components will be affected in each of these designs. The two contrasts of A corresponding to these two $A(BC)$ components and the relative loss of information on these components will be as shown in Table 2.

4. Design $q \times 3^n$

The procedure given in section 2 for the construction of the designs $q \times 3^2$ can be generalised to the case of designs $q \times 3^n$ in $2 \times 3^{n-p}$ blocks each of size $3k \times 3^p$ plots, where $3(k-1) < q \leqslant 3k$ following the procedure given by Sreenath (1965).

In the design thus constructed the affected interactions will be

(*i*) All these belonging to 'between block set' of the confounded design $(3^n, 3^{n-p-1})$ used in the construction. No information is available on them as they are completely confounded between blocks.

(*ii*) All of the type $A(Z)$, where Z is an interactian of 'between sub-blocks within blocks set' of the design $(3^n, 3^{n-p-1})$ used for the construction of the design. In each such interaction $A(Z)$ with $2(q-1)$ d.f. only 4 d.f. will be affected. These 4 d.f. are similar to those of $A(BC)$ affected in the corresponding design $q \times 3^2$ in 6 blocks of $9k$ plots each used.

The estimates of effects due to affected interaction components and the S.S. due to them can be obtained as in the case of design $q \times 3^2$.

5. Summary

In this paper methods of construction and analysis of the designs $q \times 3^2$ in 6 blocks each of $9k$ plots, where $3(k-1) < q \leqslant 3k$ for $q = 3k$ or $(3k-1)$ or $(3k-2)$ where k is a positive integer have been discussed. The extension of these procedures to the general case of designs $q \times 3^n$ has also been indicated.

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TABLE 1

Schemes of Replacement

Treatment of Combinations of Pseudo Factors		*Corresponding level of A when q is of the form*					
		$q=3k$	$q=3k-1$		$q=3k-2$		
			Type 1	*Type 2*	*Type 1*	*Type 2*	*Type 3*
X	Y	A	A	A	A	A	A
0	0	0	0	0	0	0	0
1	0	1	0	1	0	1	0
2	0	2	1	2	0	2	1
3	0	3	2	3	1	3	2
4	0	4	3	4	2	4	3
:	:	:	:	:	:	:	:
$(k-1)$	0	$(k-1)$	$(k-2)$	$(k-1)$	$(k-3)$	$(k-1)$	$(k-2)$
0	1	k	$(k-1)$	k	$(k-2)$	0	$(k-1)$
1	1	$(k+1)$	k	$(k+1)$	$(k-1)$	k	k
2	1	$(k+2)$	$(k+1)$	$(k+2)$	k	$(k+1)$	$(k+1)$
:	:	:	:	:	:	:	:
$(k-1)$	1	$(2k-1)$	$(2k-2)$	$(2k-1)$	$(2k-3)$	$(2k-2)$	$(2k-2)$
0	2	$2k$	$(2k-1)$	$2k$	$(2k-2)$	0	$(2k-1)$
1	2	$(2k+1)$	$2k$	$(2k+1)$	$(2k-1)$	$(2k-1)$	$2k$
2	2	$(2k+2)$	$(2k+1)$	$(2k+2)$	$2k$	$2k$	$(2k+1)$
:	:	:	:	:	:	:	:
$(k-3)$	2	$(3k-3)$	$(3k-4)$	$(3k-3)$	$(3k-5)$	$(3k-5)$	$(3k-4)$
$(k-2)$	2	$(3k-2)$	$(3k-3)$	$(3k-2)$	$(3k-4)$	$(3k-4)$	$(3k-3)$
$(k-1)$	2	$(3k-1)$	$(3k-2)$	0	$(3k-3)$	$(3k-3)$	$(3k-3)$

TABLE 2

Design	Type	Contrasts of A corresponding to the affected component of interaction A(BC)	Relative loss of information
$q=3k$		(i) $(a_0+a_1+\ldots+a_{k-1})-(a_{2k}+a_{2k}+a_{2k1}+\ldots+a_{3k-1})$	$\frac{1}{2}$
		(ii) $(a_0+a_1+\ldots+a_{k-1})-2(a_k+a_{k+1}+\ldots+a_{2k-1})+$	
		$(a_{2k}+a_{2k+1}+\ldots+a_{3k-1})$	$\frac{1}{2}$
$q=(3k-1)$	1	(i) $(a_{k-1}+a_k+\ldots+a_{2k-2})-(a_{2k-1}+\ldots+a_{3k-2})$	$\frac{1}{2}$
		(ii) $(a_{k-1}+a_k+\ldots+a_{3k-2})-4a_0-2(a_1+a_2+\ldots+a_{k-2})$	$\frac{(3k+4)^2}{18k(k+4)}$
	2	(i) $(a_1+a_2+\ldots+a_{k-1})-(a_{2k}+a_{2k+1}+\ldots+a_{3k-2})$	$\frac{k-1}{2k}$
		(ii) $(2a_0+a_1+a_2+\ldots+a_{k-1})-2(a_k+a_{k+1}+\ldots+a_{2k-1})$	$\frac{(3k+1)^2}{18k(k+1)}$
		$+(a_{2k}+a_{2k+1}+\ldots+a_{3k-2})$	
$q=(3k-2)$	1	(i) $(a_{k-2}+a_{k-1}+\ldots+a_{2k-3})-(a_{2k-2}+a_{2k-1}+\ldots+a_{3k-3})$	$\frac{1}{2}$
		(ii) $6a_0+2(a_1+a_2+\ldots+a_{k-3})-(a_{k-2}+a_{k-1}+\ldots+a_{3k-3})$	$\frac{(k+4)^2}{2k(k+16)}$
	2	(i) $(a_1+a_2+\ldots+a_{k-1})-(a_{2k-1}+a_{2k}+\ldots+a_{3k-3})$	$\frac{k-1}{2k}$
		(ii) $(a_1+a_2+\ldots+a_{k-1})-2(a_k+a_{k+1}+\ldots+a_{2k-2})$	$\frac{k-1}{2k}$
		$+(a_{2k-1}+a_{2k}+\ldots+a_{3k-3})$	
	3	(i) $(2a_0+a_1+a_2+\ldots+a_{k-2})-(a_{2k-1}+a_{2k}+\ldots a_{3k-4}$	$\frac{(k+2)^2}{2k(k+6)}$
		$+2a_{3k-3})$	
		(ii) $(2a_0+a_1+a_2+\ldots+a_{k-2})-2(a_{k-1}+a_k+\ldots+a_{2k-2})$	$\frac{(3k+2)^2}{18k(k+2)}$
		$+(a_{2k-1}+a_{2k}+\ldots+a_{3k-4}+2a_{3k-3})$	

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LINKING OF BIO-ASSAY CONTRASTS AND FACTORIAL CONTRASTS

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Introduction

Bliss (1940) while re-examining an experiment by Coward (1938) on the assay of vitamin D from the ash content of the femur of the rat, noticed that the estimation of relative-potency and its error can be facilitated by adopting a factorial type of analysis. Further, Bliss (1952) considered two and three dose factorial assays. However, it appears that a general link between bio-assays and factorial contrasts has not been discussed in literature. We have thus presented, here, a general link between the bio-assay and factorial contrasts which will enable us to use the traditional confounded designs for factorial experiments in bio-assays.

2. The Link

Consider a $2K$-point symmetrical parallel line (SPL) assay. The $(2K-1)$ $d.f$ between doses can be split-up into $(2K-1)$ orthogonal contrasts, each with single $d.f.$ Following notations of Finney (1952), these contrasts are $L_1, L_2, \ldots, L_{K-1}, L_1', L_2', \ldots, L'_{K-1}$ and L_p (L_m and L'_m denote the sums and differences of mth power contrasts of dose effects of the two preparations, L_p denotes the difference between the totals of the standard and test preparation effects, *i.e.*, the 'preparation contrast'). On the other hand, consider an asymmetrical factorial experiment with two factors, viz. X at two levels 0 and 1, and A at K, $(K \geqslant 2)$ levels $0,1,2,\ldots,(K-1)$. The $(2K-1)$ $d.f.$ of this $2 \times K$ factorial experiment can be split up into three components namely, (1) the main effect X with one $d.f.$, (*ii*) the main effect A with $(K-1)$ $d.f.$, and (*iii*) the interaction XA with $(K-1)$ $d.f.$ Further, using the orthogonal polynomials (Fisher and Yates, 1963, Table *XXIII*) of a set of K equally spaced levels the components A and XA can be split up into orthogonal components each with 1 $d.f.$ Thus if we denote A_i (and similarly XA_i), $i = 1,2,\ldots,(K-1)$, as the ith power contrast then the totality of $(2K-1)$ $d.f.$ of the above factorial experiment can be split up into following $(2K-1)$ orthogonal contrasts : $X, A_1, A_2, \ldots, A_{K-1}, XA_1, \ldots, XA_{K-1}$.

The correspondence between the $2K$ treatment combinations (basic treatments) of the factorial experiment and the $2K$ doses of the SPL assay can be defined as below.

$$(X_o\, a_j) \equiv s_{j+1}\,;\ (x_1\, a_j) \equiv t_{j+1},\ j=0,1,2,\ldots,(K-1), \qquad \ldots(1)$$